

CBSE Board
Class IX Mathematics
Sample Paper 1

Time: 3 hrs

Total Marks: 80

General Instructions:

1. All questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. Simplify : $(5 + \sqrt{5})(5 - \sqrt{5})$
2. Find the value of p such that $(x - 1)$ is a factor of the polynomial $x^3 + 10x^2 + px$?
3. In $\triangle ABC$, $\angle A = 100^\circ$ and $AB = AC$. Find $\angle B$?
OR
Find the angles of an isosceles triangle whose equal angles and the non-equal angles are in the ratio 3:4.
4. The cost of notebook is twice the cost of a pen. Write a linear equation in the two variables to represent this statement?
5. Find the range of data: 70, 65, 75, 71, 36, 55, 61, 62, 41, 40, 39, and 35.
OR
What is the class mark of the class interval 45-52?
6. In a parallelogram PQRS, What is the sum of $\angle R$ and $\angle S$?

Section B
(Questions 7 to 12 carry 2 marks each)

7. Simplify:
$$\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{25}{9}\right)^{-3/2}$$

OR



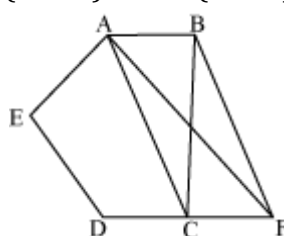
Simplify: $\left(\frac{8}{27}\right)^{-\frac{2}{3}} - \left(\frac{1}{3}\right)^{-2} - 7^0$

8. Factorise: $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

9. Where do the following points lie?

- $(-4, 0)$
- $(-10, 2)$
- $(0, 8)$
- $(10, 4)$

10. In the given figure, ABCDE is a pentagon. A line through B and parallel to AC meets DC produced at F. Show that $\text{area}(\triangle ACB) = \text{area}(\triangle ACF)$.



11. A rectangular metallic sheet has dimensions 48 cm \times 36 cm. From each corner a square of 8 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box.

OR

Find the length of the diagonal in the following cases cuboid of 11 cm by 10 cm by 2 cm.

12. Find the value of k, if $x = 1, y = 1$ is a solution of the equation $9kx + 12ky = 63$.

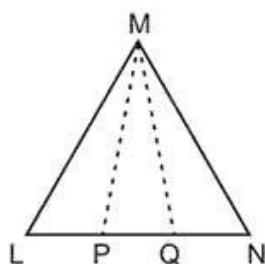
Section C

(Questions 13 to 22 carry 3 marks each)

13. Simplify: $\frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$

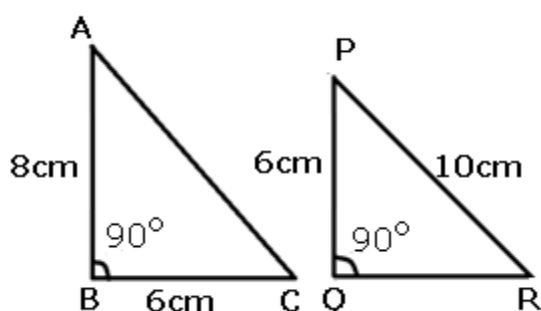
14. $(x + 2)$ is one of the factors of the polynomial $x^3 + 13x^2 + 32x + 20$. Find its remaining factors.

15. In the figure, it is given that $LM = MN$ and $LP = QN$. Prove that $\triangle LMQ \cong \triangle NMP$

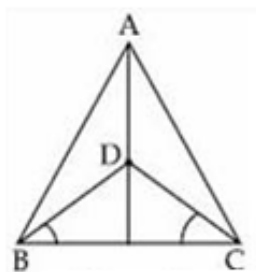


OR

The following pairs of triangles are congruent? Give reasons



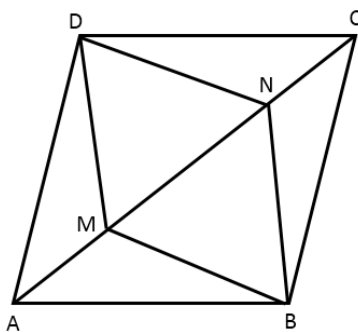
16. The polynomials $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 and R_2 . Find 'a' if $R_1 + R_2 = 0$. Factorise the polynomial.
17. In figure, $AB = AC$, D is the point in the interior of $\triangle ABC$ such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of $\triangle ABC$.



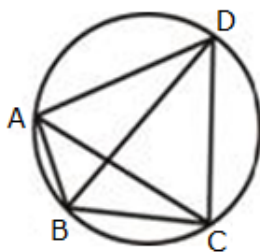
18. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is a
- queen
 - non-ace card
 - black card
19. Show that the line segments joining the mid points of the opposite sides of a quadrilateral bisect each other.

OR

Point M and N are taken on the diagonal AC of a parallelogram ABCD such that $AM = CN$. Prove that BMDN is a parallelogram.



20. In the given figure, ABCD is a cyclic quadrilateral, in which AC and BD are the diagonals. If $m\angle DBC = 55^\circ$ and $m\angle BAC = 45^\circ$, find $m\angle BCD$.



21. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move over once to level a playground. Find the area of the playground in m^2 ? $\left[\pi = \frac{22}{7}\right]$

OR

A room is 22 m long, 15 m broad and 6 m high. Find the area of its four walls and the cost of painting it including doors and windows at the rate of Rs 12 per m^2 .

22. The following observations have been arranged in the ascending order.
29, 32, 48, 50, x, x + 2, 72, 78, 84, 95
If the median of the data is 63, find the value of x.

OR

The number of goals scored by Arsenal Football Club in the English Premier League in the season 2007 was:

1, 2, 1, 3, 2, 5, 1, 6, 4, 4, 2, 3, 5, 6, 4, 2, 2, 3, 4, 1, 0, 5, 0, 5, 3, 2, 3, 4, 4, 1, 1, 2, 4, 3, 1, 4

Arrange these data in a discrete frequency distribution table and answer the following:

- What is the range of the number of goals scored by AFC?
- How many times did AFC score 3 or more than 3 goals?
- Which variate has the highest frequency?



Section D
(Questions 23 to 30 carry 4 marks each)

23. Find the value of:

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

24. How does Euclid's fifth postulate imply the existence of parallel lines? Give a mathematical proof.

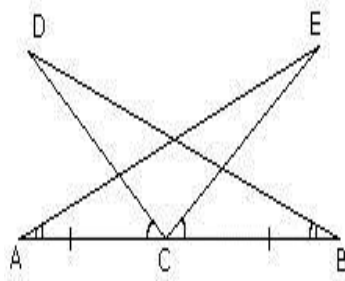
25. Simplify:

$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$$

OR

If $x - \frac{1}{x} = 5$; find (i) $x^2 + \frac{1}{x^2}$, (ii) $x^4 + \frac{1}{x^4}$

26. In the figure, if $AC = BC$, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$, then prove that $BD = AE$.



27. The volume of a cylinder is 6358 cu. cm and its height is 28 cm. Find its radius and the curved surface area.

OR

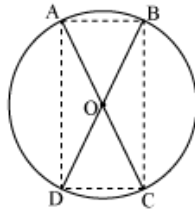
Find the cost of sinking a tube well 350 m deep, having a diameter of 4 m at the rate of Rs 16 per m^3 . Find also the cost of cementing its inner curved surface at Rs 12 per m^2

28. Construct a triangle having a perimeter of 12.5 cm and angles in the ratio of 3 : 4 : 5.
29. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

OR

AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.





30. Neha and Richa, two students of class IX of a school, together contributed Rs. 100 towards the Prime Minister's Relief Fund, to help earthquake victims. Assume Neha's contribution to be x and that of Richa to be y . Write a linear equation which this data satisfies and draw a graph of the same.

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Sample Paper 1 – Solution

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Section A

1. $(5 + \sqrt{5})(5 - \sqrt{5}) = (5)^2 - (\sqrt{5})^2 = 25 - 5 = 20$ $[\because a^2 - b^2 = (a - b)(a + b)]$

2. $p(x) = x^3 + 10x^2 + px$
 $(x - 1)$ is the factor of $p(x)$.
 $\therefore x - 1 = 0$
 $\therefore x = 1$ will satisfy $p(x)$
 $\therefore p(1) = 0$
 $\Rightarrow (1)^3 + 10(1)^2 + p(1) = 0$
 $\Rightarrow 1 + 10 + p = 0$
 $\Rightarrow p = -11$

3. Given, $AB = AC$
 $\therefore \angle B = \angle C$ (1) $(\because \text{angles opp. to equal sides are equal})$
In $\triangle ABC$, we have
 $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow \angle A + \angle B + \angle B = 180^\circ$ [From (1)]
 $\Rightarrow 100^\circ + 2\angle B = 180^\circ$ ($\because \angle A = 100^\circ$)
 $\Rightarrow 2\angle B = 80^\circ$
 $\Rightarrow \angle B = 40^\circ$

OR

The equal angles and the non-equal angle are in the ratio 3:4.
Let equal angles be $3x$ each, therefore non-equal angle is $4x$.
Angles of a triangle = 180°
 $\Rightarrow 3x + 3x + 4x = 180^\circ$
 $\Rightarrow 10x = 180^\circ$
 $\Rightarrow x = 18^\circ$
Therefore, $3x = 54^\circ$ and $4x = 72^\circ$
Angles = $54^\circ, 54^\circ$ and 72° .

4. Let, the cost of a note book = Rs. x and the cost of a pen = Rs. y
According to given statement,
 $x = 2y$
i.e. $x - 2y = 0$



5. Arranging the given data in the Ascending order:

35, 36, 39, 40, 41, 55, 61, 62, 65, 70, 71, 75

$$\therefore \text{Range} = \text{Maximum value} - \text{Minimum value} = 75 - 35 = 40$$

OR

The class mark of 45-52 is $\frac{45+52}{2} = 48.5$.

6. In a Parallelogram PQRS,

$\angle R$ and $\angle S$ are consecutive interior angles on the same side of the transversal SR.

Therefore, $m\angle R + m\angle S = 180^\circ$

Section B

- 7.

$$\begin{aligned} & \left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{25}{9}\right)^{-3/2} \\ &= \left[\left(\frac{3}{2}\right)^4\right]^{-3/4} \times \left[\left(\frac{5}{3}\right)^2\right]^{-3/2} \\ &= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{5}{3}\right)^{-3} \\ &= \left(\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^3 \\ &= \frac{2^3}{3^3} \times \frac{3^3}{5^3} = \frac{2^3}{5^3} = \frac{8}{125} \end{aligned}$$

OR

$$\begin{aligned} & \left(\frac{8}{27}\right)^{-2/3} - \left(\frac{1}{3}\right)^{-2} - 7^0 \\ &= \left(\frac{27}{8}\right)^{2/3} - 3^2 - 1 \\ &= \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - 9 - 1 \\ &= \left(\frac{3}{2}\right)^2 - 10 \\ &= \frac{9}{4} - 10 \\ &= \frac{9 - 40}{4} = \frac{-31}{4} \end{aligned}$$



8.

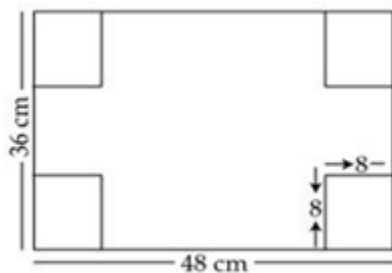
$$\begin{aligned}
 & x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x} \\
 &= \left(x^2 + \frac{1}{x^2} + 2 \right) - 2 \left(x + \frac{1}{x} \right) \\
 &= \left(x + \frac{1}{x} \right)^2 - 2 \left(x + \frac{1}{x} \right) \\
 &= \left(x + \frac{1}{x} \right) \left(x + \frac{1}{x} - 2 \right)
 \end{aligned}$$

9.

- a. Point of the form $(a, 0)$ lie on the x axis.
So, the point $(-4, 0)$ will lie on the negative side of the x axis.
- b. $(-, +)$ are the sign of the coordinate of points in the quadrant II.
So, the point $(-10, 2)$ lies in the quadrant II.
- c. Point of the form $(0, a)$ lie on the y axis.
So, the point $(0, 8)$ will lie on the positive side of y axis.
- d. $(+, +)$ are the sign of the coordinates of points in the quadrant I.
So, the point $(10, 4)$ lies in the quadrant I.

10. $\triangle ACB$ and $\triangle ACF$ lie on the same base AC and are between the same parallel lines AC and BF.
 $\therefore \text{area}(\triangle ACB) = \text{area}(\triangle ACF)$

11. Length of the box = $l = 48 - 8 - 8 = 32$ cm
Breadth of the box = $b = 36 - 8 - 8 = 20$ cm
Height = $h = 8$ cm



$$\text{Volume of the box formed} = l \times b \times h = 32 \times 20 \times 8 = 5120 \text{ cm}^3$$

OR

Given that:

Length (l) of cuboid = 11 cm

Breadth (b) of cuboid = 10 cm

Height (h) of cuboid = 2 cm

We know that,

$$\begin{aligned}\text{Length of the diagonal in a Cuboid} &= \sqrt{l^2 + b^2 + h^2} \\ &= \sqrt{11^2 + 10^2 + 2^2} \\ &= \sqrt{121 + 100 + 4} \\ &= \sqrt{221 + 4} \\ &= \sqrt{225} \\ &= 15 \text{ cm}\end{aligned}$$

12. Since $x = 1, y = 1$ is the solution of $9kx + 12ky = 1$, it will satisfy the equation.

$$\therefore 9k(1) + 12k(1) = 63$$

$$\therefore 9k + 12k = 63$$

$$\therefore 21k = 63$$

$$\therefore k = 3$$

Section C

13.

$$\begin{aligned}& \frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}} \\ &= \frac{(5^2)^{\frac{3}{2}} \times (7^3)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}} \times 7^{\frac{3}{5}}} \\ &= \frac{5^3 \times 7^{\frac{9}{5}}}{2^5 \times 2^4 \times 7^{\frac{3}{5}}} \\ &= \frac{5^3 \times 7^{\frac{9}{5}}}{2^9 \times 7^{\frac{3}{5}}} \\ &= \frac{5^3 \times 7^{\frac{6}{5}}}{2^9}\end{aligned}$$



14. Let $p(x) = x^3 + 13x^2 + 32x + 20$

$$p(-1) = -1 + 13 - 32 + 20 = -33 + 33 = 0$$

Therefore $(x + 1)$ is a factor of $p(x)$.

On dividing $p(x)$ by $(x + 1)$, we get

$$p(x) \div (x + 1) = x^2 + 12x + 20$$

Thus,

$$\begin{aligned} x^3 + 13x^2 + 32x + 20 &= (x + 1)(x^2 + 12x + 20) \\ &= (x + 1)(x^2 + 10x + 2x + 20) \\ &= (x + 1)[x(x + 10) + 2(x + 10)] \\ &= (x + 1)(x + 2)(x + 10) \end{aligned}$$

$$\text{Hence, } x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$$

15. $LM = MN$ (Given)

$$\Rightarrow \angle MLN = \angle MNL \quad (\text{angles opposite equal sides are equal})$$

$$\Rightarrow \angle MLQ = \angle MNP$$

$$LP = QN \quad (\text{Given})$$

$$\Rightarrow LP + PQ = PQ + QN \quad (\text{adding PQ on both sides})$$

$$\Rightarrow LQ = PN$$

In $\triangle LMQ$ and $\triangle NMP$

$$LM = MN$$

$$\angle MLQ = \angle MNP$$

$$LQ = PN$$

$$\triangle LMQ \cong \triangle NMP \quad (\text{SAS congruence rule})$$

OR

In $\triangle ABC$ and $\triangle PQR$

$$\angle B = \angle Q$$

$$BC = PQ$$

By Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$10^2 = 6^2 + QR^2$$

$$100 = 36 + QR^2$$

$$QR = \sqrt{100 - 36}$$

$$QR = \sqrt{64} = 8\text{cm}$$

$$AB = QR$$

Therefore,

$$\triangle ABC \cong \triangle PQR \quad (\text{SAS and RHS criteria})$$

16. When $p(x) = ax^3 + 3x^2 - 3$ is divided by $(x - 4)$, the remainder is given by

$$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$$

When $q(x) = 2x^3 - 5x + a$ is divided by $(x - 4)$, the remainder is given by

$$R_2 = 2(4)^3 - 5(4) + a = 108 + a$$

$$\text{Given: } R_1 + R_2 = 0$$

$$\Rightarrow 65a + 153 = 0$$

$$\Rightarrow a = \frac{-153}{65}$$

By hit and trial we find $x = 3$ is factor of given polynomial, as

$$2(3)^3 - 9 - 39 - 6 = 54 - 54 = 0$$

By dividing $2x^3 - x^2 - 13x - 6$ by $x - 3$ we get $2x^2 + 5x + 2$ as quotient.

Now,

$$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2 = 2x(x + 2) + 1(x + 2) = (2x + 1)(x + 2)$$

$$\text{So, } 2x^3 - x^2 - 13x - 6 = (2x + 1)(x + 2)(x - 3)$$

17. In $\triangle DCB$, $\angle DBC = \angle DCB$ (given)

$$DC = DB \text{ [Side opp. To equal } \angle\text{'s are equal].....(i)}$$

In $\triangle ABD$ and $\triangle ACD$

$$AB = AC \text{ (given)}$$

$$BD = CD \text{ [from (i)]}$$

$$AD = AD \text{ common}$$

$$\triangle ABD \cong \triangle ACD \text{ [SSS Rule]}$$

$$\angle BAD = \angle CAD \text{ (CPCT)}$$

Hence, AD is bisector of $\angle BAC$.

18. Total number of outcomes = $n(S) = 52$

- (a) Let A be the event when the card drawn is a queen.

$$\text{Total number of queen cards} = 4$$

$$\therefore n(A) = 4$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

- (b) Let B be the event when the card drawn is a non-ace card.

$$\text{Non-ace cards} = 52 - 4 = 48$$

$$\therefore n(B) = 48$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{48}{52} = \frac{12}{13}$$

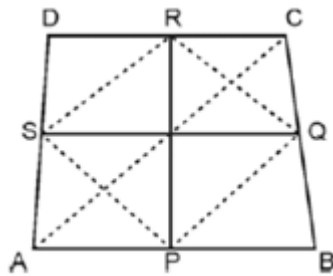
(c) Let C be the event when the card drawn is a black card.

Number of black cards = 26

$$\therefore n(C) = 26$$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

19. Let ABCD be a quadrilateral. P, Q, R, and S are the mid points of AB, BC, CD and DA respectively.



Join PQ, QR, RS and SP. Join AC.

In $\triangle DAC$, $SR \parallel AC$

And $SR = \frac{1}{2}AC$ (Mid-point theorem)

In $\triangle BAC$, $PQ \parallel AC$

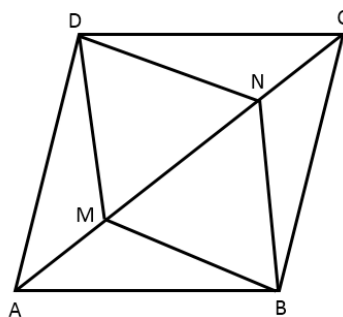
And $PQ = \frac{1}{2}AC$

Clearly, $PQ \parallel SR$ and $PQ = SR$

In quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other and hence it is a parallelogram.

Now, PR and SQ are the diagonals of PQRS and hence PR and SQ bisect each other.

OR



The diagonals of a parallelogram bisect each other.

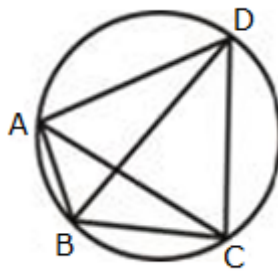
Therefore, AC and BD bisect each other.

$$\Rightarrow OA = OC$$

$$\text{But } AM = CN$$

Therefore, $OA - AM = OC - CN$
 $\Rightarrow OM = ON$
 Therefore, in quadrilateral BMDN,
 $OM = ON$ and $OD = OB$
 \Rightarrow Diagonals MN and BD bisect each other
 \Rightarrow BMDN is a parallelogram.

20. Given: ABCD is a cyclic quadrilateral in which AC and BD are diagonals.
 $m\angle DBC = 55^\circ$ and $m\angle BAC = 45^\circ$



To find: $m\angle BCD$

Proof: $m\angle CAD = m\angle DBC = 55^\circ$ (Angles in the same segment)

Therefore, $m\angle DAB = m\angle CAD + m\angle BAC$

$$= 55^\circ + 45^\circ$$

$$= 100^\circ$$

However, $m\angle DAB + m\angle BCD = 180^\circ$

(\because opposite angles of a cyclic quadrilateral)

So, $m\angle BCD = 180^\circ - 100^\circ = 80^\circ$

21. The roller is cylindrical in shape.

Height (h) of cylindrical roller = Length of roller = 120 cm

Radius (r) of the circular end of roller = $\left(\frac{84}{2}\right)$ cm = 42 cm

$$\text{C.S.A. of roller} = 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680 \text{ cm}^2$$

$$\text{Area of field} = 500 \times \text{C.S.A. of roller} = (500 \times 31680) \text{ cm}^2 = 15840000 \text{ cm}^2$$

$$\text{Area of field} = 1584 \text{ m}^2$$

OR

Given that:

Length (l) of room = 22 m

Breadth (b) of room = 15 m &

Height (h) of room = 6 m

$$\begin{aligned}\text{L.S.A of room} &= 2 \times h \times (l + b) \\ &= 2 \times 6 \times (22 + 15) \\ &= 2 \times 6 \times 37 \\ &= 444 \text{ m}^2\end{aligned}$$

\therefore Area of its 4 walls = 444 m²

Cost of painting the walls = 12 per m²

i.e., for 1 m² = Rs 12

\therefore For 444 m² = Rs 12 x 444
= Rs 5328

22. Total observations in the given data set, n = 10 (even)

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2}$$

$$\therefore 63 = \frac{\left(\frac{10}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{10}{2} + 1\right)^{\text{th}} \text{ observation}}{2}$$

$$\therefore 63 = \frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2}$$

$$\therefore 63 = \frac{(x) + (x + 2)}{2}$$

$$\therefore 63 = \frac{2x + 2}{2}$$

$$\therefore 63 = x + 1$$

$$\therefore x = 62$$

OR

The discrete frequency distribution table is as below:

No. Of goals	Tally Marks	Frequency
0		2
1		7
2		7
3		6
4		8
5		4
6		2

- (i) Maximum goals scored = 6
Minimum goals scored = 0
 \therefore Range of the goals scored = $6 - 0 = 6$
- (ii) No. Of times AFC scored 3 or more goals = $6 + 8 + 4 + 2 = 20$
- (iii) The variate which has highest frequency is 4

Section D

$$23. \frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}+\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

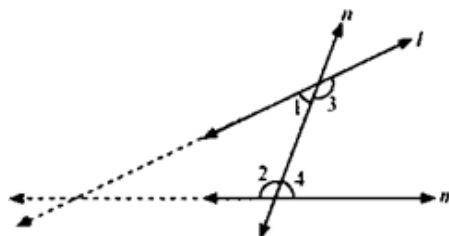
$$= 3 + \sqrt{8} - (\sqrt{8} + \sqrt{7}) + (\sqrt{7} + \sqrt{6}) - (\sqrt{6} + \sqrt{5}) + (\sqrt{5} + 2)$$

$$= 5$$

24. Euclid's 5th postulate states that:

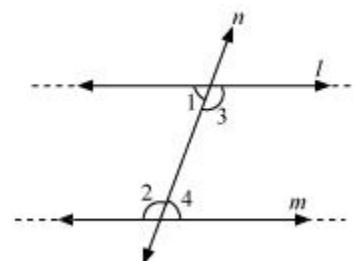
If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

This implies that if n intersects lines l and m and if $\angle 1 + \angle 2 < 180^\circ$, then $\angle 3 + \angle 4 > 180^\circ$. In that case, producing line l further will meet in the side of $\angle 1$ and $\angle 2$ which is less than 180° .



If $\angle 1 + \angle 2 = 180^\circ$, then $\angle 3 + \angle 4 = 180^\circ$

In that case, the lines l and m neither meet at the side of $\angle 1$ and $\angle 2$ nor at the side of $\angle 3$ and $\angle 4$ implying that the lines l and m will never intersect each other. Therefore, the lines are parallel.



25.

Consider
$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$$

We know that,

If $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

Now, $a^2 - b^2 + b^2 - c^2 - a^2 = 0$

And, $a - b + b - c + c - a = 0$

$$\begin{aligned}
 & \therefore \frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3} \\
 &= \frac{3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}{3(a - b)(b - c)(c - a)} \\
 &= \frac{3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)}{3(a - b)(b - c)(c - a)} \\
 &= (a + b)(b + c)(c + a)
 \end{aligned}$$

OR

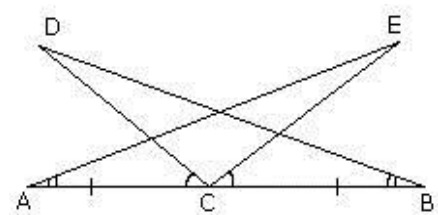
$$\begin{aligned}
 x - \frac{1}{x} &= 5 \\
 \left(x - \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} - 2 \\
 \Rightarrow 25 &= x^2 + \frac{1}{x^2} - 2 \\
 \Rightarrow x^2 + \frac{1}{x^2} &= 27
 \end{aligned}$$

$$\text{i. } x^2 + \frac{1}{x^2} = 27$$

Squaring on both sides

$$\text{ii. } x^4 + \frac{1}{x^4} + 2 = 27^2 \Rightarrow x^4 + \frac{1}{x^4} = 729 - 2 = 727$$

26. Given that $\angle DCA = \angle ECB$
 $\Rightarrow \angle DCA + \angle ECD = \angle ECB + \angle ECD$
 $\Rightarrow \angle ECA = \angle DCB$ (i)
 Now in $\triangle DBC$ and $\triangle EAC$
 $\angle DCB = \angle ECA$ [from (i)]
 $BC = AC$ (Given)
 $\angle DBC = \angle EAC$ (Given)
 $\triangle DBC \cong \triangle EAC$ (ASA Congruence)
 Therefore, $BD = AE$ (CPCT)



27. Let the radius of the cylinder be 'r' cm.

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times r^2 \times 28$$

$$\therefore 6358 = 88r^2$$

$$\therefore r^2 = \frac{6358}{88} = \frac{289}{4}$$

$$\therefore r = \frac{17}{2} \text{ cm} = 8.5 \text{ cm}$$

$$\text{Curved surface area of cylinder} = 2\pi rh = 2 \times \frac{22}{7} \times \frac{17}{2} \times 28 = 1496 \text{ cm}^2.$$

Thus, the radius of the cylinder is 8.5 cm and its curved surface area is 1796 cm².

OR

Height (h) = 350 m

Diameter = 4 m

\therefore Radius (r) = 2 m

Volume of tube well = $\pi \times r^2 \times h$

$$= \frac{22}{7} \times 2^2 \times 350$$

$$= 4400 \text{ m}^3$$

Cost of sinking the tube well = 4400 x Rs 16

$$= \text{Rs } 70400$$

L.S.A. of cylindrical tube well = $2 \times \pi \times r \times h$

$$= 2 \times \frac{22}{7} \times 2 \times 350$$

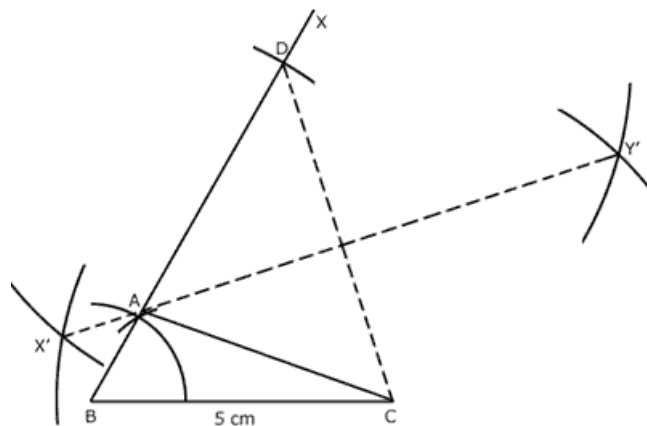
$$= 4400 \text{ m}^2$$

Cost of cementing inner curved surface of tube well = 4400 x Rs 12

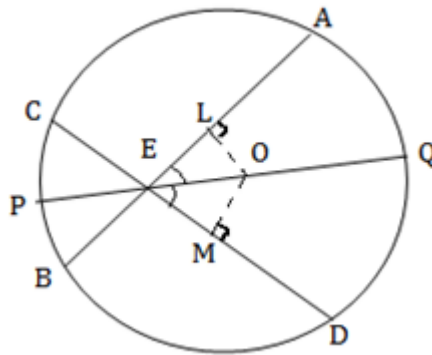
$$= \text{Rs } 52800$$

28. Steps of construction:

- Draw BC = 5 cm
- Draw $\angle CBX = 60^\circ$ and cut off BD = 7.7 cm.
- Join CD and draw its perpendicular bisector meeting BD at A.
- Join AC. $\triangle ABC$ is the required triangle.



29. Given that AB and CD are two chords of a circle with centre O, intersecting at a point E.



PQ is the diameter through E, such that $\angle AEQ = \angle DEQ$.

To prove that $AB = CD$.

Draw perpendiculars OL and OM on chords AB and CD respectively.

Now, $m\angle LOE = 180^\circ - 90^\circ - m\angle LEO$... [Angle sum property of a triangle]
 $= 90^\circ - m\angle LEO$

$$\Rightarrow m\angle LOE = 90^\circ - m\angle AEQ$$

$$\Rightarrow m\angle LOE = 90^\circ - m\angle DEQ$$

$$\Rightarrow m\angle LOE = 90^\circ - m\angle MEQ$$

$$\Rightarrow \angle LOE = \angle MOE$$

In $\triangle OLE$ and $\triangle OME$,

$$\angle LEO = \angle MEO$$

$$\angle LOE = \angle MOE$$

$$EO = EO$$

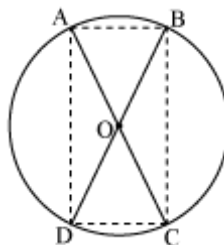
$$\triangle OLE \cong \triangle OME$$

$$OL = OM$$

Therefore, chords AB and CD are equidistant from the centre.

Hence, $AB = CD$

OR



In $\triangle AOB$ and $\triangle COD$

$OA = OC$ (given)

$OB = OD$ (given)

$\angle AOB = \angle COD$ (vertically opposite angles)

$\triangle AOB \cong \triangle COD$ (SAS congruence rule)

$AB = CD$ (by CPCT)

Similarly, we can prove $\triangle AOD \cong \triangle COB$

$\therefore AD = CB$ (by CPCT)

Since in quadrilateral ABCD opposite sides are equal in length.

Hence, ABCD is a parallelogram.

We know that opposite angles of a parallelogram are equal

$\therefore \angle A = \angle C$

But $m\angle A + m\angle C = 180^\circ$ (ABCD is a cyclic quadrilateral)

$\Rightarrow m\angle A + m\angle A = 180^\circ$

$\Rightarrow 2m\angle A = 180^\circ$

$\Rightarrow m\angle A = 90^\circ$

As ABCD is a parallelogram and one of its interior angles is 90° , so it is a rectangle.

$\angle A$ is the angle subtended by chord BD. And as $m\angle A = 90^\circ$, so BD should be the diameter of the circle. Similarly AC is the diameter of the circle.

30. According to the given condition,

$$x + y = 100 \quad \dots(i)$$

Now, put the value $x = 0$ in equation (i).

$$0 + y = 100 \Rightarrow y = 100$$

The solution is (0, 100)

Putting the value $x = 50$ in equation (i), we get

$$50 + y = 100 \Rightarrow y = 100 - 50 \Rightarrow y = 50.$$

The solution is (50, 50).

Put the value $x = 100$ in equation (i).

$$100 + y = 100,$$

$$y = 100 - 100 \Rightarrow y = 0.$$

The solution is (100, 0).

x	0	50	100
y	100	50	0

Now, plot the points (0, 100), (50, 50), (100, 0) and draw lines passing through the points.



